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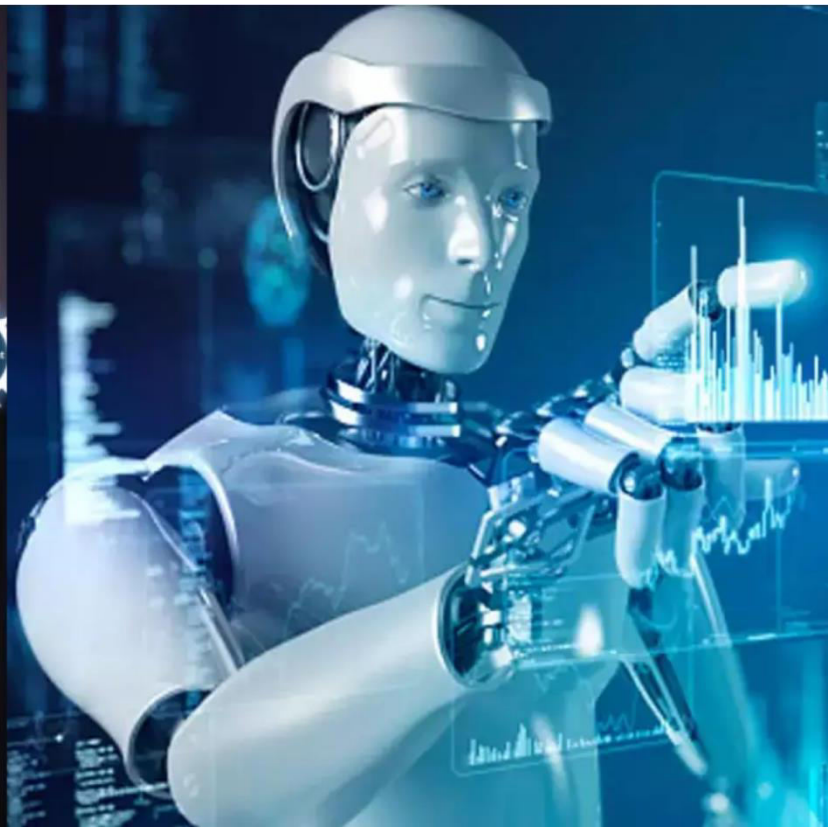
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Inversion Theorem of New Modelling of Offset Fractional Fourier Transform

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ABSRTACT: The fractional Fourier transform (FrFT) is an important mathematical tool that generalizes the classical Fourier transform by introducing a continuous order parameter. It provides an effective framework for analyzing signals in joint time frequency domains and has found applications in signal processing, optical systems, and mathematical physics. The Offset fractional Fourier transform (FrFT) extends the conventional fractional Fourier transform by incorporating additional Offset parameters, thereby providing greater flexibility in the representation and analysis of signals. In this paper, we investigate the inversion properties of the New Modelling of Offset fractional Fourier transform and establish an inversion theorem under suitable mathematical conditions. The proposed analysis focuses on recovering the original function from its corresponding Offset fractional Fourier transform through an appropriate inverse integral representation. The role of the fractional order and Offset parameters in the reconstruction process is examined, and the relationship between the FrFT and the conventional FrFT is discussed. The validity of the inversion result is established through mathematical arguments, providing a theoretical foundation for the reconstruction of functions in the new modelling of Offset fractional Fourier domain. The results contribute to the further development of the mathematical theory of the Offset FrFT and may be useful in applications involving signal reconstruction and time frequency analysis.

KEYWORDS: Offset Fractional Fourier Transform, Fractional Fourier Transform, Inversion Theorem, Integral Transform, Inverse Transform.

I. INTRODUCTION

Integral transforms play a fundamental role in mathematical analysis and have been extensively employed in solving differential equations, studying physical phenomena, and analyzing signals and systems. Among the various integral transforms, the classical Fourier transform is one of the most widely used mathematical tools. It converts a function from the time domain into the frequency domain, thereby providing an effective approach to studying the spectral characteristics of signals. However, the conventional Fourier transform does not always provide sufficient flexibility for analyzing signals whose frequency characteristics vary with time. This limitation has motivated the development of generalized forms of the Fourier transform.

The Fractional Fourier Transform (FrFT) is an important extension of the classical Fourier transform in which the transformation is characterized by a continuous order parameter. The concept of fractional order Fourier transformation was introduced by Namias [1] in the context of mathematical physics and quantum mechanics. Subsequently, Almeida [2] investigated its significance in time frequency representations and demonstrated that the FrFT can be interpreted geometrically as a rotation in the time frequency plane. This interpretation provides a useful mathematical framework for studying non-stationary signals and their frequency distributions. Further developments and applications of the FrFT have been discussed extensively by Ozaktas, Zalevsky, and Kutay [4].

The Offset fractional Fourier transform represents a further generalization of the fractional Fourier transform through the introduction of additional Offset parameters. These parameters provide additional degrees of freedom in the transformation process and allow a more flexible representation of signals in the fractional Fourier domain. In contrast to the conventional FrFT, the Offset FrFT incorporates additional phase and displacement characteristics through its transformation kernel. Consequently, the Offset FrFT provides a generalized mathematical framework for investigating signal transformations and reconstruction problems.

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The mathematical properties of fractional integral transforms are closely connected with their inversion formulas. An inversion theorem is particularly important because it establishes the conditions under which the original function can be recovered from its transformed representation. The existence of an appropriate inverse transformation ensures that the transformation preserves the essential information contained in the original function. In the context of the FrFT, inversion and related operational properties have been investigated in the literature [1], [2], [4]. Generalized inversion formulas for multidimensional fractional Fourier transforms have also been studied by V. D. Sharma [5]. More recently, Bahri et al. [3] investigated several properties of the Offset Fractional Fourier transform and presented an inverse representation along with convolution, product, and sampling results. Their work provides an important contribution to the development of the mathematical theory of the Offset FrFT. Nevertheless, the investigation of inversion properties under specific analytical assumptions, together with their rigorous mathematical justification and possible extensions, remains an important subject of study.

Motivated by these developments, the present paper is devoted to the study of the inversion theorem for the New Modelling of Offset fractional Fourier transform. The main objective is to establish a mathematically justified inversion result that enables the recovery of the original function from its Offset FrFT representation. The analysis considers the influence of the fractional order and the Offset parameters on the inverse transformation and examines the conditions required for the validity of the proposed result. Particular attention is given to the integral representation of the transform and the mathematical steps involved in recovering the original function. By considering the integral representation of the new modelling of Offset FrFT and the role of its associated parameters, an inversion theorem is formulated and established under suitable assumptions.

II. THE TESTING FUNCTION SPACE E(R^n)

An infinitely differentiable complex valued function ϕ on R^n belongs to E(R^n), if for each compact set $I \subset S_{a,b}$, where

$$S_{a,b} = \{x, y : x, y \in R^n, |x| \le a, |y| \le b, a > 0, b > 0\},$$

$\gamma_{E_{m,n}}(\phi) = \sup_{x,y \in I} \left| D_{x,y}^{m,n} \phi(x,y) \right| < \infty$, $m, n = 0,1,2, \ldots \ldots$

Thus, E(R^n) will denote the space of all $\phi \in$ E(R^n) with compact support contained in $S_{a,b}$.

2.1 THEOREM

To prove for each $x, y \in R^n$ and $0 < \alpha \le \frac{\pi}{2}$, the function $K_\alpha(x, u-\eta, y, p)$ is belongs to E(R^n) as function of x and y, where

$$K_\alpha(x, u-\eta, y, p) = \sqrt{\frac{1 - i\cot\alpha}{2\pi}}\, y^{p-1} e^{iu\tau} e^{\frac{i}{2\sin\alpha}\{[x^2 + (u-\eta)^2]\cos\alpha - 2[x(u-\eta)]\}}$$

Proof: Consider

$$\gamma_{E_{m,n}}[K_\alpha(x, u-\eta, y, p)] = \sup_{x,y \in I} \left| D_{x,y}^{m,n} K_\alpha(x, u-\eta, y, p) \right|$$

$$= \sup_{x,y \in I} \left| D_{x,y}^{m,n} \sqrt{\frac{1 - i\cot\alpha}{2\pi}}\, y^{p-1} e^{i(u\tau)} e^{\frac{i}{2\sin\alpha}\{[x^2 + (u-\eta)^2]\cos\alpha - 2[x(u-\eta)]\}} \right|$$

$$= \sup_{x,y \in I} \left| C_{1\alpha} e^{iu\tau} \sum_{k=0}^{m} C_m \; C_\alpha\big(x\cos\alpha - 2(u-\eta)\big)^{m-2k} e^{(m-k)q} (p-n)!\, y^{p-n-1} \right|$$

$< \infty$,

for any fixed $x, y \in R^n$ and any fixed m, n, $0 < \alpha \le \frac{\pi}{2}$.

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hus, function $K_\alpha(x,u-\eta,y,p) \in E(R^n)$.

III. GENERALIZED NEW MODELLING OF OFFSET FRACTIONAL FOURIER TRANSFORM.

The distributional New Modelling of Offset fractional Fourier transform of $f(x,y) \in E^*(R^n)$ is defined by,

$$F_\alpha^{\tau,\eta}\{f(x,y)\}(u,p) = F_\alpha(u,p) = \langle f(x,y),\ K_\alpha(x,u-\eta,y,p))\rangle, \qquad \ldots\ldots\ldots\ldots \quad (1)$$

Where

$$K_\alpha(x,u-\eta,y,p) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, y^{p-1} e^{iu\tau} e^{\frac{i}{2sin\alpha}\{[x^2+(u-\eta)^2\,]cos\alpha - 2[x(u-\eta)]\}} \qquad \ldots. (2)$$

$$= C_{1\alpha} y^{p-1} e^{i(u\tau)} e^{iC_{2\alpha}\{[x^2+(u-\eta)^2\,]cos\alpha - 2[x(u-\eta)]\}},$$

where $C_{1\alpha} = \sqrt{\frac{1-icot\alpha}{2\pi}}$, $C_{2\alpha} = \frac{1}{2sin\alpha}$

Here right hand side of equation (1) is meaningful because according to above theorem equation (1), $K_\alpha(x,u-\eta,y,p) \in E$ and $f \in E^*$.
Moreover, we say that f is a Offset fractional Fourier-Mellin transformable, if it is a member of E^*, the dual space of E.

IV. INVERSION OF GENERALIZED NEW MODELLING OF OFFSET FRACTIONAL FOURIER TRANSFORM

In this section, inversion formula of generalized new modelling of Offset fractional Fourier transform is obtained by using inversion formula of conventional Fourier transform.

4.1 Inversion formula

The distributional new modelling of Offset fractional Fourier transform of $f(x,y) \in E^*(R^n)$ is defined by,

$$MF_\alpha^{\tau,\eta}\{f(x,y)\}(u,p) = F_\alpha(u,p) = \langle f(x,y),\ K_\alpha(x,u-\eta,y,p))\rangle, \qquad \ldots (4.1)$$

where

$$K_\alpha(x,u-\eta,y,p) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, y^{p-1} e^{iu\tau} e^{\frac{i}{2sin\alpha}\{[x^2+(u-\eta)^2\,]cos\alpha - 2[x(u-\eta)]\}}$$

then by inversion it is possible to recover $f(x,y)$ by means of the inversion formula

$$f(x,y) = \int_{-\infty}^{\infty}\int_{0}^{\infty} F_\alpha(u,p)\widetilde{K}_\alpha(x,u-\eta,y,p)dudp,$$

where $\widetilde{K}_\alpha(x,u-\eta,y,p) = \frac{e^{-i(u\tau)}}{(2\pi)^{3/2}sin\,\alpha\sqrt{1-icot\alpha}} e^{\frac{-i}{2}\{[x^2+(u-\eta)^2\,]cot\alpha - 2[x(u-\eta)]cosec\alpha\}} y^{p-1}$

Proof: Consider

$$F_\alpha^{\tau,\eta}\{f(x,y)\}(u,p) = F_\alpha(u,p)$$

$$= \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{i(u\tau)} \int_{-\infty}^{\infty}\int_{0}^{\infty} f(x,y)\, y^{p-1} e^{\frac{i}{2sin\alpha}\{[x^2+(u-\eta)^2\,]cos\alpha - 2[x(u-\eta)]\}} dxdy$$

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$$= C_{1\alpha} e^{i(u\tau)} \int_{-\infty}^{\infty} \int_{0}^{\infty} f(x,y) y^{p-1} \, e^{iC_{2\alpha}\{[x^2+(u-\eta)^2]cos\alpha - 2[x(u-\eta)]\}} dxdy,$$

where $C_{1\alpha} = \sqrt{\frac{1-icot\alpha}{2\pi}}$, $C_{2\alpha} = \frac{1}{2sin\alpha}$

$$= e^{iC_{2\alpha}[(u-\eta)^2]cos\alpha} \int_{-\infty}^{\infty} \int_{0}^{\infty} C_{1\alpha} e^{i(u\tau)} f(x,y) y^{p-1} e^{iC_{2\alpha}\{[x^2]cos\alpha - 2[x(u-\eta)]\}} dxdy,$$

$$F_{\alpha}(u,p) = e^{iC_{2\alpha}[(u-\eta)^2]cos\alpha} \int_{-\infty}^{\infty} \int_{0}^{\infty} C_{1\alpha} e^{i(u\tau)} f(x,y) y^{p-1} \, e^{\frac{i}{2}[x^2]cot\alpha} e^{-i[x(u-\eta)]cosec\alpha} \, dxdy$$

$$e^{-iC_{2\alpha}[(u-\eta)^2]cos\alpha} F_{\alpha}(u,p) = \int_{-\infty}^{\infty} \int_{0}^{\infty} g(x,y) \, y^{p-1} e^{-i[(u-\eta)cosec\alpha]x} \, dxdy$$

where $g(x,y) = C_{1\alpha} e^{i(u\tau)}) e^{\frac{i}{2}(x^2)cot\alpha} f(x,y)$

$$e^{-iC_{2\alpha}[(u-\eta)^2]cos\alpha} F_{\alpha}(u,p) = \int_{-\infty}^{\infty} \int_{0}^{\infty} g(x,y) y^{p-1} \, e^{-iAx} \, dxdy, \qquad \ldots (4.2)$$

where $A = (u-\eta)cosec\alpha$
$dA = du \, cosec\alpha$

$$e^{-iC_{2\alpha}[(u-\eta)^2]cos\alpha} F_{\alpha}(u,p) = F[g(x,y)](A)$$

$$e^{-iC_{2\alpha}[(u-\eta)^2]cos\alpha} F_{\alpha}\left(\frac{A}{cosec\alpha} + \eta, \quad p\right) = F[g(x,y)](A,p) \qquad \ldots (4.3)$$

The right hand side is the Fourier-Mellin transform of $g(x,y)$ with argument A. Invoking the Fourier-Mellin inversion we can write

$$g(x,y) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{0}^{\infty} G(A,p) e^{i(Ax)} y^{p-1} dAdy \qquad \ldots (4.4)$$

Now putting the value of $g(x,y)$

$$C_{1\alpha} e^{i(u\tau)} e^{\frac{i}{2}(x^2)cot\alpha} f(x,y) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{0}^{\infty} G(A,p) e^{i(Ax)} y^{p-1} dAdy$$

$$= \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-iC_{2\alpha}[(u-\eta)^2]cos\alpha} F_{\alpha}\left(\frac{A}{cosec\alpha} + \eta, p\right)$$

$$f(x,y) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{0}^{\infty} (C_{1\alpha})^{-1} e^{-i(u\tau)} e^{\frac{-i}{2}(x^2)cot\alpha} e^{\frac{-i}{2sin\alpha}[(u-\eta)^2]cos\alpha} \, e^{i[x(u-\eta)]cosec\alpha} cosec\alpha y^{p-1} \, du \, dp$$

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$$e^{i[x(u-\eta)]cosec\alpha}\, y^{p-1} F_\alpha(u,p) cosec\ \alpha\ dudp$$

$$= \frac{cosec\ \alpha}{4\pi^2} \int_{-\infty}^{\infty}\int_{0}^{\infty} (C_{1\alpha})^{-1}\, e^{-i(u\tau)}$$

$$e^{\frac{-i}{2}\{[x^2+(u-\eta)^2\ ]cot\alpha - 2[x(u-\eta)]cosec\alpha\}} y^{p-1} F_\alpha(u,p) dudp$$

$$= \frac{cosec\ \alpha}{4\pi^{\ 2}} e^{-i(u\tau)} \int_{-\infty}^{\infty}\int_{0}^{\infty} \left(\sqrt{\frac{1-icot\alpha}{2\pi}}\right)^{-1}$$

$$e^{\frac{-i}{2}\{[x^2+(u-\eta)^2\ ]cot\alpha - 2[x(u-\eta)]cosec\alpha\}} y^{p-1} F_\alpha(u,p) dudp$$

$$= \frac{cosec\ \alpha}{4\pi^2} e^{-i(u\tau)} \int_{-\infty}^{\infty}\int_{0}^{\infty} \frac{(2\pi)^{1/2}}{\sqrt{1-icot\alpha}}$$

$$e^{\frac{-i}{2}\{[x^2+(u-\eta)^2\ ]cot\alpha - 2[x(u-\eta)]cosec\alpha\}} y^{p-1} F_\alpha(u,p) dudp$$

$$= \frac{e^{-i(u\tau)}}{(2\pi)^{3/2} sin\ \alpha\sqrt{1-icot\alpha}} \int_{-\infty}^{\infty}\int_{0}^{\infty} e^{\frac{-i}{2}\{[x^2+(u-\eta)^2\ ]cot\alpha - 2[x(u-\eta)]cosec\alpha\}}\ y^{p-1} \qquad F_\alpha(u,v) dudv$$

$$F_\alpha(u,p) dudp$$

$$f(x,y) = \int_{-\infty}^{\infty}\int_{0}^{\infty} F_\alpha\,(u,p) \tilde{K}_\alpha(x,u-\eta,y,p) dudp,$$

where $\tilde{K}_\alpha(x,u-\eta,y,p) = \frac{e^{-i(u\tau)}}{(2\pi)^{3/2} sin\ \alpha\sqrt{1-icot\alpha}} e^{\frac{-i}{2}\{[x^2+(u-\eta)^2\ ]cot\alpha - 2[x(u-\eta)]cosec\alpha\}} y^{p-1}$ ... (4.5)

4.2 Inversion theorem on the space of generalized function

To prove the inversion theorem for generalized new modelling of Offset fractional Fourier transform on space of generalized function as per Zemannian [8] and Bhosale [9]. Inversion theorem requires two lemmas which have been proved in this section.

4.2.1 Lemma-I

Let $F_\alpha^{\tau,\eta}\{f(x,y)\}(u,p) = F_\alpha(u,p)$, for $0 < \alpha \le \frac{\pi}{2}$ and $supp\ f \subset S_{a,b}$,

$$S_{a,b} = \{x,y: x,y \epsilon R^n, |x| \le a, |y| \le b, a > 0, b > 0\}.$$

For $\phi(x,y) \in E(R^n)$,

$$\text{Set } \varphi(u,p) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \phi(x,y)\, \tilde{K}_\alpha(x,u-\eta,y,p) dxdy$$

Then for any fixed real number $s,t, -\infty < s < \infty, -\infty < t < \infty$

$$\int_{-s}^{s}\int_{-t}^{t} \varphi(u,p)\, \langle f(x,y), K_\alpha(x,u-\eta,y,p)\rangle d\mu d\vartheta$$

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$$= \langle f(x,y), \int_{-s}^{s}\int_{-t}^{t} \varphi(u,p)\, K_\alpha(x, u-\eta, y, p)\rangle\, d\mu d\vartheta, \qquad \ldots (4.6)$$

where $u = \sigma + i\mu,\ p = \rho + i\vartheta,\ u, p \in C^n, \sigma, \rho,$ are fixed, $\sigma_1 < Re\, x < \sigma_2$, $\rho_1 < Re\, y < \rho_2$.

Proof: The case $\phi(x,y) = 0$ is trivial. So assume that $\phi(x,y) \neq 0$.
It can be easily seen that

$$\int_{-s}^{s}\int_{-t}^{t} \varphi(u,p)\, K_\alpha(x, u-\eta, y, p) d\mu d\vartheta,$$

where $u = \sigma + i\mu,\ p = \rho + i\vartheta,$ is a $C^\infty -$ function of λ, δ and it is belongs to E.

Hence RHS of equation (4.6) is meaningful.
To prove the equality, we construct the Riemann-Sums for this integral and write

$$\int_{-s}^{s}\int_{-t}^{t} \langle f(\lambda,\delta), K_\alpha(\lambda, u-\eta, \delta, p)\rangle\, \varphi(u,p) d\mu d\vartheta$$

$$= \lim_{\substack{q\to\infty \\ l\to\infty}} \sum_{q=-r}^{r-1}\sum_{k=-l}^{l-1} \langle f(\lambda,\delta), K_\alpha\big(\lambda, (\sigma + i\mu_{q,r} - \eta), \delta, (\rho + i\vartheta_{k,l} - p)\big)\rangle$$

$$\varphi\big(\sigma + i\mu_{q,r}, \rho + i\vartheta_{k,l}\big)\Delta\mu_{q,r}\Delta\vartheta_{k,l}$$

$$= \lim_{\substack{q\to\infty \\ l\to\infty}} \langle f(\lambda,\delta), \sum_{q=-r}^{r-1}\sum_{k=-l}^{l-1} K_\alpha\big(\lambda, (\sigma + i\mu_{q,r} - \eta), \delta, (\rho + i\vartheta_{k,l} - p)\big)\rangle$$

$$\varphi\big(\sigma + i\mu_{q,r}, \rho + i\vartheta_{k,l}\big)\Delta\mu_{q,r}\Delta\vartheta_{k,l}$$

we show that last summation converges in E to the integral on the RHS of (4.6). Taking the operator $D_{\lambda,\delta}^{m,n}$ within the integral and summation signs, which is easily justified, we get

$$\gamma_{I_{m,n}} \left\{ \sum_{q=-r}^{r-1}\sum_{k=-l}^{l-1} K_\alpha\big(\lambda, (\sigma + i\mu_{q,r} - \eta), \delta, (\rho + i\vartheta_{k,l} - p)\big)\varphi\big(\sigma + i\mu_{q,r}, \rho + i\vartheta_{k,l}\big) \Delta\mu_{q,r}\Delta\vartheta_{k,l} - \int_{-s}^{s}\int_{-t}^{t} \varphi(u,v)\, K_\alpha(\lambda, u-\eta, \delta, v-p) d\mu d\vartheta \right\}$$

$$= \sup_{\substack{\lambda\in I \\ \delta\in I}} \left| \sum_{q=-r}^{r-1}\sum_{k=-l}^{l-1} D_{\lambda,\delta}^{m,n} K_\alpha\big(\lambda, (\sigma + i\mu_{p,q} - \eta), \delta, (\rho + i\vartheta_{k,l} - p)\big)\varphi\big(\sigma + i\mu_{p,q}, \rho + i\vartheta_{k,l}\big) \Delta\mu_{q,r}\Delta\vartheta_{k,l} - \int_{-s}^{s}\int_{-t}^{t} \varphi(u,v)\, K_\alpha(\lambda, u-\eta, \delta, p) \right|$$

As

$$\lim_{\substack{q\to\infty \\ l\to\infty}} \sum_{q=-r}^{r-1}\sum_{k=-l}^{l-1} D_{\lambda,\delta}^{m,n}\, K_\alpha\big(\lambda, \sigma + i\mu_{p,q} - \eta, \delta, \rho + i\vartheta_{k,l} - p\big)\Delta\mu_{q,r}\Delta\vartheta_{k,l}$$

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$$\varphi(\sigma + i\mu_{p,q}, \rho + i\vartheta) = \int_{-s}^{s}\int_{-t}^{t} \varphi(u,v)\, K_\alpha(\lambda, u-\eta, \delta, p) d\mu d\vartheta$$

for all $\lambda, \delta \in I$.

It thus follows that for every q and l, the summation is a member of E and it converges in E to the integral on the RHS of (4.6).

4.2.2 Lemma-II

For $\phi(x,y) \in E(R^n)$, set $\varphi(u,p)$ as in lemma 4.2.1 above for $u, p \in C^n$, λ, δ restricted to compact subset of R^n, then

$$M_{s,t}(\lambda,\delta) = \frac{1}{4\pi^2}\int_{-s}^{s}\int_{-t}^{t} \varphi(u,p) K_\alpha(\lambda, u-\eta, \delta, p)\, d\mu d\vartheta$$

$$= \frac{1}{4\pi^2}\int_{-s}^{s}\int_{-t}^{t} K_\alpha(\lambda, u-\eta, \delta, p) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \phi(x,y)$$

$$\widetilde{K}_\alpha(x, u-\eta, y, p) dx\, dy\, d\mu\, d\vartheta$$

Converges in E to $\phi(\lambda,\delta)$ as $s \to \infty, t \to \infty$. ... (4.7)

Proof: We shall show that,

$M_{s,t}(\lambda,\delta) \to \phi(\lambda,\delta)$ in E as $s \to \infty, t \to \infty$, that is to show

$$\gamma_{I_{m,n}}\left[M_{s,t}(\lambda,\delta) - \phi(\lambda,\delta)\right] = \sup_{\substack{\lambda\in I \\ \delta \in J}} \left| D_\lambda^m D_\delta^n \left[M_{s,t}(\lambda,\delta) - \phi(\lambda,\delta)\right]\right| \to 0$$

as $s \to \infty, t \to \infty$.

We note that for $m = 0, n = 0$.

$$\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{0}^{\infty} K_\alpha(\lambda, u-\eta, \delta, p) \int_{-\infty}^{\infty}\int_{0}^{\infty} \phi(x,y)\, \widetilde{K}_\alpha(x, u-\eta, y, p) dx\, dy\, d\mu\, d\vartheta$$

$$= \phi(\lambda,\delta)$$

This is to say that $\lim_{\substack{q\to\infty \\ l\to\infty}} M_{s,t}(\lambda,\delta) = \phi(\lambda,\delta)$.

Since the integrand is a C^∞-function of λ, δ and $\phi(x, y \in E(R^n)$, we can repeatedly differentiate under the integral sign in (4.7) and integrals are uniformly convergent, we have

$$\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} D_{\lambda,\delta}^{m,n} K_\alpha(\lambda, u-\eta, \delta, p)$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \phi(x,y)\, \widetilde{K}_\alpha(x, u-\eta, y, p) dx\, dy\, d\mu\, d\vartheta = \phi(\lambda,\delta)$$

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for all $\lambda, \delta \in I$.

4.2.3 Inversion theorem

Let $f(x,y) \in E(R^n)$, $0 < \alpha \leq \frac{\pi}{2}$ and $suppf \subset S_{a,b}$, where

$$S_{a,b} = \{x,y: x,y \epsilon R^n, |x| \leq a, |y| \leq b, a > 0, b > 0\} \text{ and}$$

let $F_\alpha(u,p)$ be the distributional new modelling of Offset fractional Fourier transform of f as defined by

$$F_\alpha^{\tau,\eta}\{f(x,y)\}(u,p) = F_\alpha(u,p) = \langle f(x,y), K_\alpha(x,u-\eta,y,p)\rangle,$$

where $K_\alpha(x,u-\eta,y,p) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{i(u\tau)}$

$$e^{\frac{i}{2sin\alpha}\{[(x^2+(u-\eta)^2)]cos\alpha - 2[(x(u-\eta))]\}} y^{p-1}$$

Then for each $\phi(x,y) \in E(R^n)$, we have

$$\lim_{\substack{q\to\infty \\ K\to\infty}} \langle \frac{1}{4\pi^2} \int_{-q}^{q}\int_{-K}^{K} \widetilde{K}_\alpha(x,u-\eta,y,p)\, F_\alpha(u,p) du\, dp\,,\ \phi(x,y)\rangle = \langle f(x,y), \phi(x,y)\rangle,$$

where $\widetilde{K}_\alpha(x,u-\eta,y,p) = \sqrt{\frac{1+icot\alpha}{2\pi}}\, e^{-i(u\tau)}$

$$e^{\frac{-i}{2sin\alpha}\{[x^2+(u-\eta)^2]cos\alpha - 2[x(u-\eta)]\}} y^{p-1}$$

Proof: Let $\phi(x,y) \in E(R^n)$, we shall show that,

$\langle \frac{1}{4\pi^2} \int_{-s}^{s}\int_{-t}^{t} \widetilde{K}_\alpha(x,u-\eta,y,p)\, F_\alpha(u,p) d\mu d\vartheta\,,\ \phi(x,y)\rangle$ tends to $\langle f(x,y), \phi(x,y)\rangle$ as $q \to \infty, r \to \infty$. ... (4.8)

From the analyticity of $F_\alpha(u,p)$ on C^n and the fact that $\phi(x,y)$ has compact support in R, it follows that the left hand side expression in (4.8) is merely a repeated integral with respect to x,y and u,p and the integral in (4.8) is a continuous function of x,y as the closed bounded domain of the integration. Therefore, we write (4.8) as

$$= \frac{1}{4\pi^2} \int_{-\infty}^{\infty}\int_{0}^{\infty} \phi(x,y) \int_{-s}^{s}\int_{-t}^{t} \widetilde{K}_\alpha(x,u-\eta,y,p)\, F_\alpha(u,p) dx dy d\mu d\vartheta$$

$$= \frac{1}{4\pi^2} \int_{-\infty}^{\infty}\int_{0}^{\infty} \phi(x,y) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \widetilde{K}_\alpha(x,u-\eta,y,p)$$

$$\langle f(\lambda,\delta),\ K_\alpha(\lambda,u-\eta,\delta,p)\rangle dx dy d\mu d\vartheta$$

$$= \frac{1}{4\pi^2} \int_{-s}^{s}\int_{-t}^{t} \langle\, f(\lambda,\delta),\ K_\alpha(x,u-\eta,y,p)\rangle$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \phi(x,y)\ \widetilde{K}_\alpha(x,u-\eta,y,p) dx dy d\mu d\vartheta \qquad \text{... (4.9)}$$

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Left hand side of (4.6)

$$= \frac{1}{4\pi^2} \int_{-s}^{s} \int_{-t}^{t} \langle f(\lambda,\delta),\ \widetilde{K}_\alpha(\lambda, u-\eta, \delta, p)\rangle\, \varphi(u,p)\, d\mu d\vartheta,$$

where $\varphi(u,p)$ is an in lemma 4.2.1

$$= \langle f(\lambda,\delta), \frac{1}{4\pi^2} \int_{-s}^{s} \int_{-t}^{t} \widetilde{K}_\alpha(x, u-\eta, y, p)\, \varphi(u,p) d\mu d\vartheta \rangle$$

$$= \langle f(x,y),\ \phi(x,y)\rangle \text{ as } s \to \infty, t \to \infty. \quad \text{... (by lemma 4.2.2 and (4.9))}$$

This completes the proof.

Uniqueness theorem

If $F_\alpha^{\tau,\eta}\{f(x,y)\}(u,p) = F_\alpha(u,p)$ and $F_\alpha^{\tau,\eta}\{g(x,y)\}(u,p) = G_\alpha(u,p)$for $0 < \alpha \leq \frac{\pi}{2}$, $suppf \subset S_{a,b}$, $suppg \subset S_{a,b}$,

where

$S_{a,b} = \{x, y: x, y \epsilon R^n, |x| \leq a, |y| \leq b, a > 0, b > 0$. If $F_\alpha(u,p) = G_\alpha(u,p)$then $f = g$ in the sense of equality in $E^*(I)$.

Proof: By inversion theorem

$f(x,y) - g(x,y)$

$$= \lim_{\substack{q\to\infty \\ K\to\infty}} \frac{1}{4\pi^2} \int_{-q}^{q} \int_{-K}^{K} \widetilde{K}_\alpha(x, u-\eta, y, p)[F_\alpha(u,p) - G_\alpha(u,p)]dudp\ = 0$$

Thus, $f\ (x,y) = g(x,y)$in $E^*(I)$.

V. CONCLUSION

In this paper, the inversion theorem of the New Modelling of Offset fractional Fourier transform have been proved. The study focuses on establishing a mathematical framework for recovering an original function from its corresponding new modelling of Offset fractional Fourier transform. The obtained result provides a theoretical basis for the reconstruction of functions in the offset fractional Fourier domain. It is demonstrating how the additional offset parameters influence the transformation and its inverse representation. The inversion result contributes to the broader mathematical understanding of generalized fractional integral transforms.

The significance of the present investigation lies in providing a systematic mathematical treatment of the inverse transformation and establishing the conditions necessary for recovering the original function. Moreover, the established mathematical framework may find applications in signal reconstruction, optical signal processing, and other areas where fractional domain representations are employed.

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